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A Generalized model for the design of MEMS electrostatic actuators

Anurekha Sharma

Abstract— The MEMS sensing and actuating devices make use of the electrostatic principle. A number of MEMS structures make use of deformation and/or pull in of the diaphragms and beams for performing sensing and actuation. The electrostatic actuator may have air as a dielectric or may have an intervening layer of dielectric or multilayered dielectric along with air-gap. For any of these structures the design specification is given in terms of the pull-in Voltage (V_{Pull}) and desired value of capacitance. The paper presents a semi-analytical technique for determining the generalized expression for pull-in voltage, critical distance and capacitance at pull-in for all the three types of structures. It has been found that for an actuator with only a single intervening layer of dielectric, the dielectric constant greater than 10 is not desirable, thus SiO_2, Si_3N_4 are the only choices. However, if multilayer structure is used, high K materials can be used. For given structural parameters and material properties, it has been found that a figure of merit can be given as $V_{Pull}^2 C_{Pull}^3 = 9.94\times10^{-20} Da^2$, a is the half side length of the square diaphragm, D is flexural rigidity .Using these equations one can find the pull-in voltage as well as predict the capacitance for the actuator, for the given structural and material properties. One can also decide the thickness of the dielectric for a given air-gap to achieve a specified value of pull-in.

Index Terms— MEMS, Electrostatic actuator, Pull-in, Multilayered dielectric, Model, Actuation, Intellisuite, Simulation

1 Introduction

The MEMS sensing and actuating devices make use of the electrostatic principle. A number of MEMS structures make use of deformation and/or pull in of the diaphragms and beams for performing sensing and actuation. The diaphragms may be circular, square or rectangular. Square diaphragms find use in numerous applications like electrostatic valve actuator for high-pressure applications [1], polysilicon micromirrors [2], silicon capacitive microphone [3], micropumps [4], bio-medical applications [5] as well as in touch-mode pressure sensors [6]. When these devices are used as actuators, the critical design parameters are pull-in voltage (V_{Pull}) [7], critical distance (w_{cr}) and desired value of capacitance. Each of these quantities can be expressed in term of the structural parameters and material properties of the diaphragm. It is well known that for an actuator with air gap, pull-in occurs at a critical displacement equal to one-third of the gap between the electrodes[7]. Employing an intervening layer of dielectric or multilayered layered dielectric on the fixed electrode can increase the pull-in voltage and critical distance. The ratio of thickness of the dielectric medium to its dielectric constant denoted by deff determines the pull-in voltage and capacitance. The paper presents a semi-analytical technique for determining a generalized expression of pull-in voltage, critical distance and capacitance at pull-in for an electrostatic actuator with only air as a dielectric or along with a single layer of dielectric or multilayered dielectric for a square diaphragm

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2 Theory of Small Deflection of Diaphragm

The typical structure of the electrostatic transducer with a single layer of dielectric is as shown in Fig. 1. According to the classic theory of plate bending, a small-deflection case is defined as a case where the displacement of the middle plane is small compared to the plate thickness [6]. The relation between the lateral load q and the deflection w(x,y) at any point (x,y) of a plate in the case of small deflection is given by :

$$\Delta\Delta w(x,y)=\frac{q}{D} \tag{1}$$

where w is deflection of the plate mid-surface , $\Delta = \frac{\partial^2}{\partial x^2}+\frac{\partial^2}{\partial y^2}$,

q is the normal load per unit area, D is flexural rigidity= $Eh^3/12(1-\upsilon^2)$, E is Young's modulus of the diaphragm material, h is the thickness of the diaphragm and ? is the Poisson's ratio.

In the presence of applied pressure and applied voltage, (1) is modified as :

$$\Delta\Delta w=\frac{P+P_{el}}{D} \tag{2}$$

where, Pel is the electrostatic pressure and P is the mechanical pressure. The electrostatic pressure Pel is given as :

$$P_{el}=\frac{\varepsilon_0\varepsilon_r}{2d^2}V^2 \tag{3}$$

where, d is distance between the plates, is relative permittivity of the medium or the dielectric constant of the medium and is permittivity of free space.

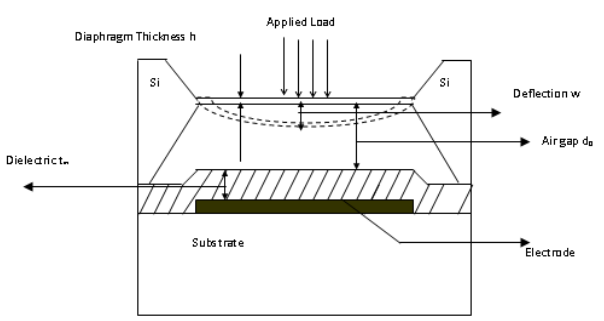

Fig 1. Cross-sectional view of electrostatic transducer with a single layer of dielectric

The diaphragm deflection w(x, y) with air as dielectric is given by considering that the distance d between the plates changes to due to the displacement w of the diaphragm in the presence applied pressure and voltage as shown in figure 1. Therefore, substituting (3) in (2) for air gap , the equation becomes :

$$D\Delta\Delta w = P + \varepsilon_0 \varepsilon_a \frac{V^2}{2(d_0 - w)^2} \tag{4}$$

For diaphragm with intervening layer of dielectric

$$D\Delta\Delta w = P + \varepsilon_i^2 \varepsilon_a \varepsilon_0 \frac{V^2}{2(\varepsilon_a t_m + \varepsilon_i d_0 - w))^2} \tag{5}$$

For a layered dielectric with two dielectric layers of thickness d_1 and d_2 with dielectric constants ε_1 andε_2 respectively (Fig. 2) , the equation becomes :

$$D\Delta\Delta w = P + \varepsilon_a \varepsilon_1^2 \varepsilon_2^2 \varepsilon_0 \frac{V^2}{2(\varepsilon_a \varepsilon_1 d_2 + \varepsilon_a \varepsilon_2 d_3 + \varepsilon_1 \varepsilon_2 (d_0 - w))^2} \tag{6}$$

where d0 is zero pressure gap, is the dielectric constant of the air, is the dielectric constant of the insulator,is permittivity of free space = 8.85x10-12 F/m and is the thickness of the insulator.

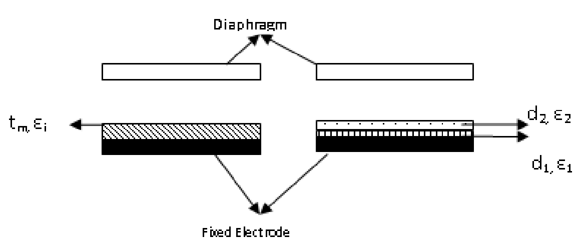

Fig 2. Representation of single and multi-layered dielectric on a fixed electrode

As ε_a =1, Eq. (5), Eq. (6) and Eq.(7) can be combined and rewritten in a generalized form as :

$$D\Delta\Delta w = P + \varepsilon_0 \frac{V^2}{2(d_{eff} + (d_0 - w))^2} \tag{7}$$

where $d_{eff} = t_m/\varepsilon_i$ (for a single layer of dielectric) and deff = d1/ε1 + d2/ε2 (in case of a layered dielectric), in general

$d_{eff} = d_1/\varepsilon_1 + d_2/\varepsilon_2 +$ -------------+ d_n/ε_n (for a n-layered dielectric)

3 SEMI-ANALYTICAL MODEL

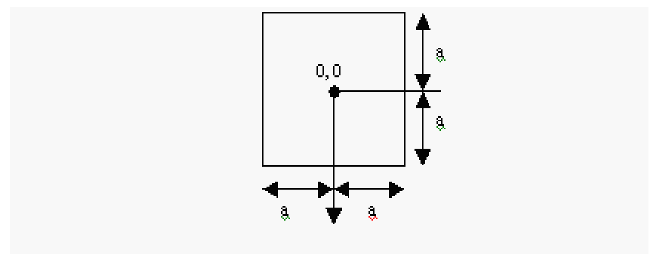

Fig 3. Co-ordinate system and dimensions for a square diaphragm

The boundary conditions for the square diaphragm with clamped edges are as follows

The deflection w(x, y) = 0 at x = , y = , = 0, = 0 at x =, y =, x = 0 and at y = 0

where 2a is the side length of the diaphragm (fig. 2)The approximate solution that satisfies the above given boundary conditions is :

$$w(x,y) = \lambda (x^2 - a^2)^2 (y^2 - a^2)^2 \tag{8}$$

where λ is a function which depends on voltage V, diaphragm thickness h , Young's modulus E , Poisson's ratio υ and dielectric constant ε_r of the medium. The equation governing λ for air as dielectric is calculated by substituting eq. (8) in eq. (7) and is given by [8] :

$$\begin{aligned} &\lambda^3 + \lambda^2\left(-\frac{2d_0}{a^8} - \frac{49P}{2304a^4 D} - \frac{2d_{eff}}{a^8}\right) + \\ &\lambda\left(\frac{d_0^2}{a^{16}} + \frac{49d_0 P}{1152a^{12} D} + \frac{2d_0 d_{eff}}{a^{16}} + \frac{49\, Pd_{eff}}{1152a^{12}\varepsilon_i D} + \frac{d_{eff}^2}{a^{16}}\right) - \\ &\left(\frac{49d_0^2 P}{2304a^{20} D} + \frac{49d_0 d_{eff} P}{1152a^{20}\, D} + \frac{49Pd_{eff}^2}{2304a^{20} D} + \frac{49\varepsilon_0\, V^2}{4608a^{20} D}\right) = 0 \end{aligned} \tag{9}$$

For determining the pull-in voltage the real root of λ in (9) is evaluated at x=0 and y=0. The resulting expression for deflection is differentiated with respect to voltage and solved for pull-in voltage by equating to zero. The critical distance is got by substituting the pull-in voltage for voltage V in expression for deflection.

For air as dielectric the expression for the pull-in voltage (V_{pull}) and critical distance (w_{cr}) is given as:

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$$\text{Vpull} = \frac{32}{7a^2}\sqrt{\frac{2}{3}\frac{(d_0+d_{eff})^3 D}{\varepsilon_0}} \tag{10}$$

$$\text{wcr} = \frac{d_0}{3} + \frac{d_{eff}}{3} \tag{11}$$

The Capacitance is given as :

$$C = \int_{-a}^{+a}\int_{-a}^{+a} \frac{\varepsilon_0 dxdy}{d_{eff} + (d_0 - w)} \tag{12}$$

The above relations have been verified by simulation with Intellisuite and the results are in agreement with the simulated ones [8]. At Pull-in

$$V_{pull} = \frac{32}{7a^2}\sqrt{\frac{2}{3}\frac{(3w_{cr})^3 D}{\varepsilon_0}} \tag{13}$$

$C_{pull} \sim \dfrac{\varepsilon_0 A}{d_{eff} + (d_0 - \frac{w_{cr}}{3})}$ A is the area of cross-section of the diaphragm

The figure of merit is given by

$$V_{Pull}^2 C_{Pull}^3 = 9.94\times10^{-20} Da^2 \text{ m}^3 \tag{14}$$

Eq. (14) along with (10), (11) and (12) can be used to design a transducer for a given pull-in voltage or capacitance.

Table I gives the comparison between the simulated and calculated values of pull-in voltage and capacitance for a silicon diaphragm (E=130 GPa) of half side length a=661µm, thickness h=5.1µm, air-gap d_0=2µm

TABLE 1.

COMPARISON BETWEEN THE CALCULATED AND SIMULATED PARAMETERS FOR AIR AS DIELECTRIC

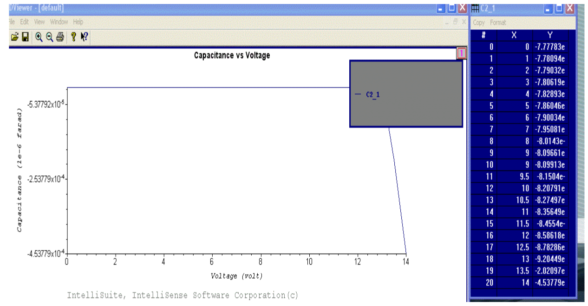
Figure of Merit (Calculated)	Pull-in Voltage (V)		Capacitance at pull-in	
$9.94\times10^{-20} Da^2 (m^3)$	Cal.	Sim.	Cal.	Sim
6.68×10^{-32}	10.2	12.5	8.6 pF	8.78pF

Figure 4a and 4b shows the simulated results for the above case.

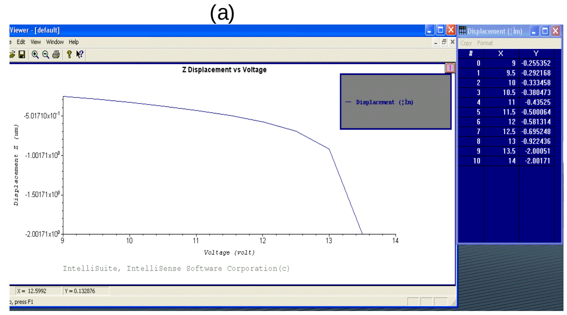
A. Effect of layered –dielectric on the pull-in voltage and Capacitance of electrostatic transducer

The zero-voltage capacitance is given as

$$C_0 = \frac{\varepsilon_0 A}{d_{eff} + d_0}$$



(a)



(b)

Fig 4. (a) Pull-in Voltage from displacement versus voltage (b) Capacitance versus voltage

From (10) it can be seen that the pull-in voltage varies as $(d_{eff})^{1.5}$ whereas capacitance varies inversely as d_{eff}. So, as the dielectric constant of the medium increases for a given thickness of dielectric Vpull decreases more rapidly as compared to increase in capacitance. The fig. 5 shows that V_{pull} saturates for values of dielectric constant >10, so the obvious choice of dielectric when a single layer of dielectric is used is SiO_2 or Si_3N_4. However, if one wants to have more flexibility in design then the layered dielectric is a good proposition.

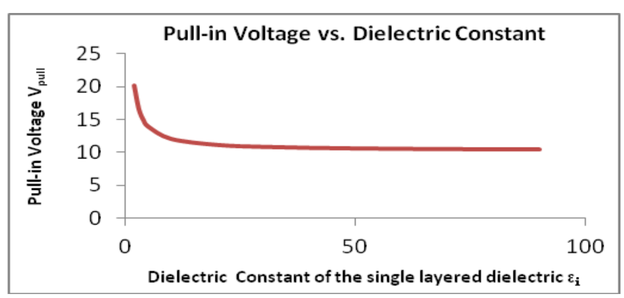


Fig 5. Effect of the dielectric constant on the pull-in voltage (diaphragm dimensions a=661 µm, h= 5.1 µm ,t_m=2.3 µm E= 130 GPa, d_0=2.0 µm)

Let us now define the normalized pull-in voltage and normalized capacitance in the following manner

$$V_{nor} = \frac{V_{pull\text{-}die}}{V_{pull\text{-}air}} = \frac{\frac{32}{7a^2}\sqrt{\frac{2}{3}\frac{(d_0+d_{eff})^3 D}{\varepsilon_0}}}{\frac{32}{7a^2}\sqrt{\frac{2}{3}\frac{d_0^3 \; D}{\varepsilon_0}}}$$

, where $V_{pull\text{-}air}$ is the pull-in voltage for air as dielectric and $V_{pull\text{-}die}$ is the pull-in voltage with an intervening layer of dielectric or layered dielectric .

$$V_{nor} = \left(1+\frac{d_{eff}}{d_0}\right)^{3/2} \tag{15}$$

For a single layer of dielectric :

$$V_{norsingle} = \left(1+\frac{t_m}{\varepsilon_i d_0}\right)^{3/2} \tag{16}$$

and for a layered dielectric with two dielectric materials normalized pull-in voltage is :

$$V_{nordouble} = \left(1+\frac{d_1}{\varepsilon_1 d_0}+\frac{d_2}{\varepsilon_2 d_0}\right)^{3/2} \tag{17}$$

The capacitance is normalized with respect to capacitance with air-gap and is given as :

$$C_{nor} = \frac{C_{die}}{C_{air}} = \frac{\frac{\varepsilon_0 A}{d_0+d_{eff}}}{\frac{\varepsilon_0 A}{d_0}} = \frac{1}{1+\frac{d_{eff}}{d_0}}$$

For single dielectric, normalized capacitance is :

$$C_{norsingle} = \frac{1}{1+\frac{t_m}{\varepsilon_i d_0}} \tag{18}$$

$$C_{nordouble} = \frac{1}{1+\frac{d_1}{\varepsilon_1 d_0}+\frac{d_2}{\varepsilon_2 d_0}} \tag{19}$$

For attaining higher value of capacitance with a layered dielectric with same overall thickness as the single layer i.e. $d_1+d_2= t_m$ requires $C_{nordouble} > C_{norsingle}$

Therefore :

$$\frac{t_m}{\varepsilon_i} > \frac{d_1}{\varepsilon_1}+\frac{d_2}{\varepsilon_2} \tag{20}$$

Let $\varepsilon_2 > \varepsilon_1$, then the following cases arise

Case I When the lower layer is of the same material as the single layer dielectric, then $\varepsilon_1= \varepsilon_i$, then Eq. (20) becomes

$$\frac{t_m}{\varepsilon_i} > \frac{d_1}{\varepsilon_i}+\frac{d_2}{\varepsilon_2}$$

Now, let us consider that = pt_m (where $0<p<1$) , therefore $d_2 = (1-p)t_m$ also let $\varepsilon_2= K\varepsilon_i$ where K is any number greater than zero. The above equation becomes

$$\frac{t_m}{\varepsilon_i} > \frac{pt_m}{\varepsilon_i}+\frac{(1\text{-}p)t_m}{K\varepsilon_i} \Rightarrow 1 > p+\frac{(1\text{-}p)}{K}$$

For the above inequality to be satisfied, K>1. The fig 6. shows the variation of normailsed pull-in voltage and normalized capacitance for a single layer of dielectric and a layered dielectric for d_0 = 2.0µm, t_m=2.3 µm for different values of ε_i starting from 2. The graphs clearly indicate that significant increase in capacitance can be brought about if the layer with lower dielectric constant is thinner

Case II $\varepsilon_1=K_1\varepsilon_i$ and $\varepsilon_2= K_2\varepsilon_1$, then if $C_{nor2} > C_{nor1}$ we have

$$1 > \frac{1}{K_1}\left(p+\frac{(1\text{-}p)}{K_2}\right)$$

$K_2 \neq 1$, as it would mean single layer of dielectric, thus the above inequality would be satisfied for K_1 >1 and K_2 >1. $K_2 = K_1$ is also admissible.

The fig. 7 shows the variation of normalized capacitance as well as pull-in voltage for the single layer as well as double layer. Table II compares the pull-in voltage for air, single layered dielectric and double layered dielectric and Figures 8a and 8b give the simulated results of the capacitance for single layered dielectric SiO_2 (ε_i =3.9) and multilayered dielectric consisting of lower layer of SiO_2 (ε_1 =3.9) and upper layer of Si_3N_4 (ε_2=7.5) and p=.25 i.e. thickness of SiO_2 is 0.575µm and that of Si_3N_4 is 1.725µm.

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4 Conclusions

Following conclusions have been drawn on the basis of the above discussion.

1. Design of the electrostatic transducer presents a trade-off between pull-in voltage and achievable capacitance. With the increase in the dielectric constant of the single layer, the pull-in voltage decreases more rapidly as compared to increase in capacitance.
2. The maximum value of zero-gap capacitance in presence of single as well multilayered dielectric is limited by the air gap. Similarly, the minimum value of the pull-in in both cases is limited by the air-gap.
3. The capacitance can be increased marginally if instead of employing a single layer, one employs a double layer
4. A figure of merit is found, which can be used to design electrostatic transducer for a given pull-in voltage irrespective of the single, multi or no dielectric. This figure of merit serves as a useful tool to find the geometric dimensions of the device.

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Using these equations one can find the pull-in voltage as well

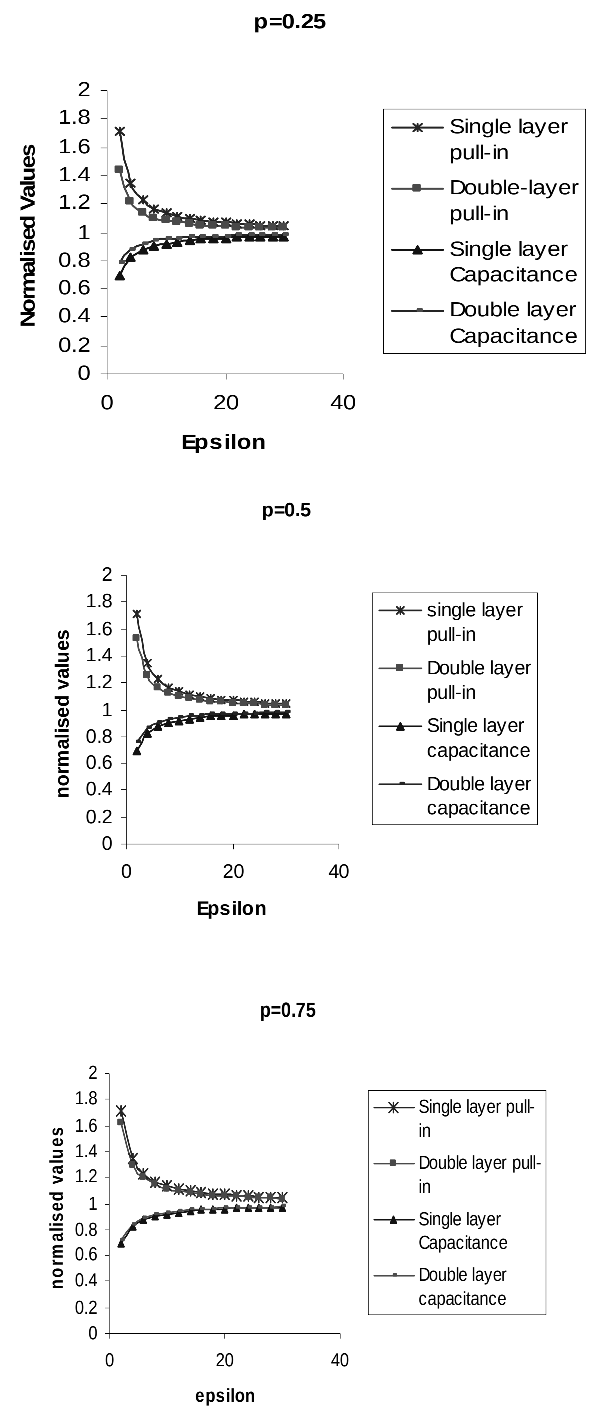


Fig 6. Normalised Pull-in Voltage and Pull-in Capacitance for different thickness of lower layer of dielectric (a) 0.25 times the single layer (b) 0.5 times the single layer (c) 0.75 times the single layer

as predict the capacitance for the actuator, for the given structural and material properties. One can also decide the thickness of the dielectric for a given air-gap to achieve a particular value of pull-in.

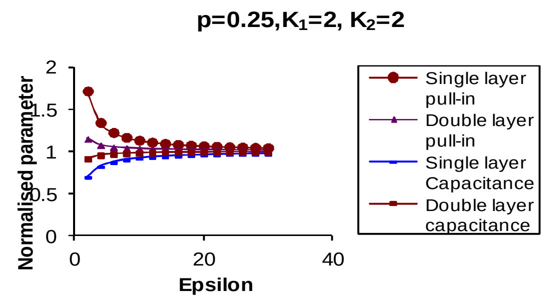
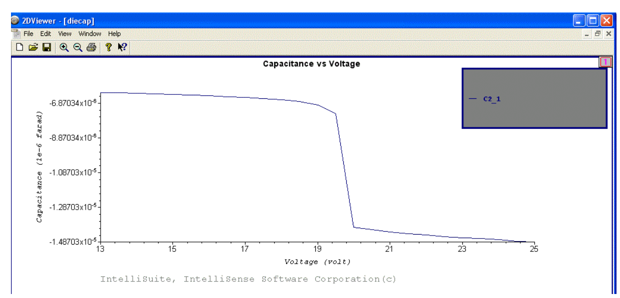


Fig 7. Variation of Normalised capacitance and pull-in voltage

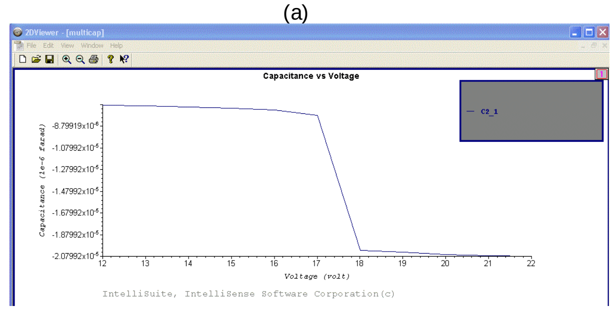
TABLE 2.

COMPARISON OF PULL-IN VOLTAGE AND CAPACITANCE

Case	Pull-in Voltage (V)		Capacitance at Pull-in (pF)	
	Cal.	Simulated	Cal.	Simulated
Air as dielectric	10.2	12.5	8.6	8.78
Single layer dielectric	15.0	19.0	6.6	6.96
Double Layered Dielectric	13.2	17.0	7.2	7.8

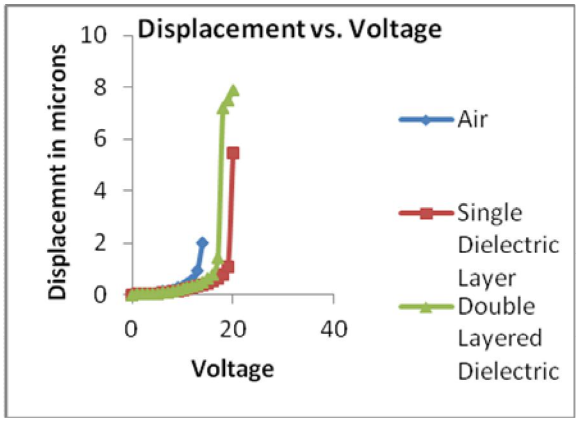


(a)

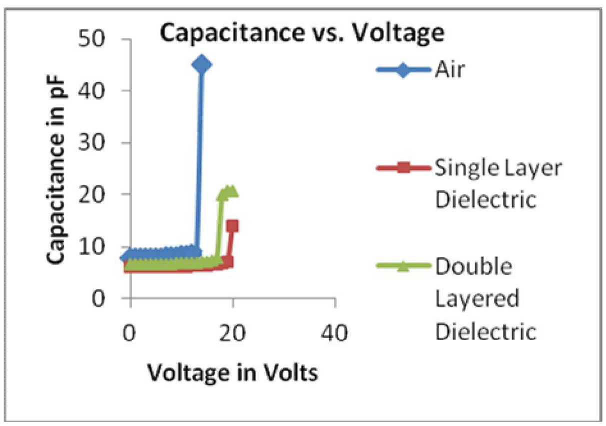


(b)

Fig 8. Simulated Capacitance vs. Voltage for (a) single layered dielectric (b) for a double layered dielectric



(a)



(b)

Fig. 9 Comparison of (a) Displacement for three cases (b) Capacitance for three cases

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